Modeling optimal deployment of smart home devices and battery system using MILP

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***Abstract*—A home energy management system (HEM), which is an integral part of smart grid, can not only allow more comfortable control over smart home appliances but also could reduce electricity bill. With the integration of battery system, a household will become prosumer with the possibility to optimize its own energy use during a certain time period and sell its own excess of electricity on the market. This paper proposes an algorithm to schedule smart home appliances and battery system, by taking into account preferences of an end user as well as the system constraints. The target function is to minimize the total energy costs. Finally, the results of the proposed model in terms of cost savings have been analyzed on three test cases for a typical day.**

***Index Terms*--GAMS; MILP; smart home; HEM; EMS**

I. NOMENCLATURE

The notation used in this paper is explained below.

Constants:

ε system inertia (0.96)
μ efficiency of the air conditioner (2.5)
A thermal conductivity [kW/°C]
T_{MIN}^{in} lowest indoor temperature [°C]
T_{MAX}^{in} highest indoor temperature [°C]
T_{IDEAL}^{in} ideal indoor temperature [°C]
Eb^{min} minimum energy stored in battery [kWh]
Eb^{max} maximum energy stored in battery [kWh]
γ_{bc} efficiencies for battery charge (94%)
γ_{bd} efficiencies for battery discharge (97%)
Pbc_t^{max} maximal charge limit of the battery (1.5 kW)
Pbd_t^{max} maximal discharge limit of the battery (0.9 kW)

Variables:

PX_t^{buy} energy bought at the market price C_t in the particular time horizon t [kWh]
PX_t^{sell} energy sold at the market price C_t in the particular time horizon t [kWh]
Pbc_t energy charged by the battery in the particular time horizon t [kWh]
Pbd_t energy discharged out of the battery in the time period t [kWh]
$d_{i,t}$ consumption of a certain home energy appliance i in the time period t [kWh]
$u_{i,t}$ binary variable that determines on/off state of an appliance i in the time interval t
$v_{i,t}$ binary variable that determines start-up status of appliance i in time interval t
$w_{i,t}$ binary variable that determines shut-down status of appliance i in time interval t
T_t^{in} indoor air temperature in time interval t [°C]
bc_t binary variable that determines battery charging mode
bd_t binary variable that determines battery discharging mode

Parameters:

C_t market price of electricity in the particular time horizon t [cEUR/kWh]
MUT_i minimal consecutive time intervals during which home appliance i needs to run once it is started [h]
MDT_i minimal consecutive time intervals during which home appliance i needs to stay shut down once it has been disconnected [h]
$Tmin_i$ minimal powered time of a certain energy appliance i during the entire time horizon T [h]
$Tstart_i$ earliest moment of time when a certain energy appliance i could be started [h]
$Tend_i$ latest moment of time when a certain energy appliance i needs to finish its work [h]
$d_{i,MAX}$ maximal consumption of a certain home energy appliance i [kWh]
Eb_t energy stored in battery [kWh]
T_t^{out} outdoor air temperature in time interval t [°C]

Sets:

t set which describes time interval t of a day (quarter of hours) (1…T); T = 96
i set of home appliances i which are to be optimized (1…N); N=9

978-1-5386-1953-7/17/$31.00 ©2017 IEEE

II. INTRODUCTION

Traditional energy systems are built on the concept that non-flexible load needs to be covered with the flexible energy supply. End consumers, such as households, conclude a contract with an energy supplier that delivers the energy at the flat rate. It means that the risk related to change of energy needs, due to the consumer behavior expressed over a load profile, is shifted to the commercial balancing group in which this particular consumer could be found. Logically, this uncertainty is already priced in and included in the flat rate given to a household. With the introduction of demand-side response and a new role of aggregators in the European electricity market, the approach of non-flexible load will change as the demand would become more elastic and would respond to the market-based signals. According to [1], Member States of European Union shall ensure that final customers are entitled to generate, store, consume and sell self-generated electricity in all organized markets, either individually or through aggregators. Those, so called active consumers, will be encouraged to participate alongside generators in a non-discriminatory manner in all organized markets.

A. Background

Previous research has been carried out on scheduling and optimization of residential load. In [2], a system that minimizes residential electricity costs by shifting demand over a daily forecast price cycle has been evaluated. The effectiveness of the proposed model in terms of cost savings by considering three appliances and four pricing schemes has been analyzed. Simulation of demand response actions on an algorithm deployed for HEM system has been described in [3]. With its application, it is possible to perform load curtailments, while considering customers' preferences. The authors in [4] performed optimal scheduling of energy consumption by adjusting operation tasks based on different electricity tariffs and forecasted renewable energy output. In [5], minimization of electricity cost by scheduling home devices using Mixed Integer Linear Programming (MILP) is proposed, with the goal to provide insight into tariff design. In [6], authors use MATLAB/SIMULINK to solve a similar optimization problem.

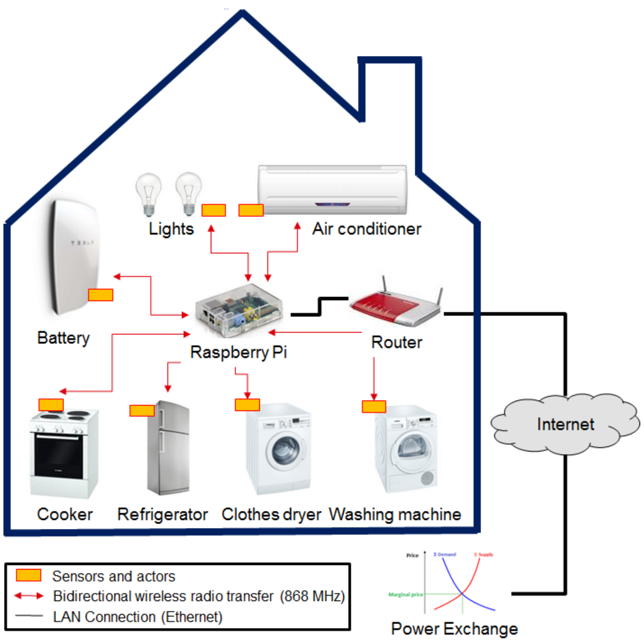

Fig. 1. Architecture of a smart home solution

B. Motivation

For the purpose of adopting end energy usage based on the market signals, a cost-effective radio signal controlled system has been developed. Using this system, we don't only perform an analysis of electricity costs reduction by shifting periods when home appliances are active, but also consider a possibility to optimize energy consumption profiles by using additionally a battery storage system. On top of that, customer's preferences, as well as additional constraints such as temperature dependent consumption and/or maximal power that could be withdrawn from the distribution network, have been taken into account. Unit commitment problem for the home appliances has been defined and solved by minimizing the total energy costs in the predefined time period of one day (96 x 15 minutes), using MILP.

C. Organisation of the paper

In section III, hardware and software architecture of the smart home system has been explained, followed by the description of target function for optimization, along with its constraints. Section IV includes information on input parameters and results of three test cases that have been analyzed. At the end, the main findings are summarized in section V.

III. SMART HOME SOLUTION

A. Hardware and software architecture

A smart home solution plays an important role in energy transition as it enables consumers to better control their home appliances. To ensure flawless transmission of control and measurement signals among different home appliances and a central computational and controlling unit (CCCU), a system which consists of Raspberry Pi, CUL modules and Homematic/FS20 devices has been selected (Fig. 1). Raspberry Pi is a powerful computer with ARM-processor on which Raspbian system (a free operating system based on Debian) could be installed, along with server software called FHEM ("Freundliche Hausautomatisierung und Energie-Messung"). This Perl-based software actually enables connection of the different home devices, such as sensors and actors and establishment of a connection among them. For the development of smart home solution, wireless 868 MHz radio transmission (Homematic and FS20) have been selected. The main difference among them is the working mode: Homematic namely works bidirectional which ensure more reliable operation as a confirmation of successfully performed actions is returned back from actors towards CCCU. The communication signal between actors/sensors on one side and Raspberry Pi server software FHEM on the other side is ensured by using wireless dongle from Busware (CUL868). The dongle is a transceiver (CC1101) that has extra 8-bit Atmel microcontroller (ATMega32U4) which sends and receives signals of 868 MHz band. Although the system is very convenient when it comes to commands which need to be received, forwarded and proceeded in the time span of few seconds, both radio protocols which are deployed need to fulfil legal obligations valid for 868 MHz frequency area. Those include limitations of the maximal broadcast time per device which is given as 1 second in 100 seconds. This limitation includes also time needed to establish a secure connection and to send confirmation signals, which is especially valid for Homematic protocol. This could be a limitation in case that large amount of data in higher frequency needs to be transmitted. Nevertheless, in the applied concept such limitations are not of importance, as data exchange is performed in a period of quarter-of-hour. In such a case, even an optimization of energy usage during the actual day would be feasible, either over the intraday markets or over a real-time price signal mechanisms.

As mentioned, Raspberry Pi plays a role of the CCCU that receives the results of optimization problem. Those results are stored in MySQL database which communicates with the FHEM server. Additionally, all important measurements from the different devices, such as indoor and outdoor temperature, consumption, voltage, switch state (on/off) are also stored in the database and aggregated on a level of quarter of an hour. The devices could additionally communicate to each other, but in the implemented architecture all data exchange among them is performed over CCCU that acts as a common gateway.

Fig. 2. FTUI FHEM – Graphical User Interface (GUI)

In total there are nine (9) home appliances, out of which eight (8) are modeled as operating with the constant power if they are scheduled to consume energy in the certain period of time. Such appliances, which are denoted as a "type 1" include, among others, cooker, light bulbs and washing machine. On the other side, consumption of some devices such as air conditioner is modelled as a linear dependence on temperature, and not only on indoor, but also on the outdoor temperature. Graphical User Interface (GUI), which has been installed on the CCCU, allows the end user to get an overview of the main parameters, such as status of device and its scheduled consumption as well as the actual measured consumption. Access over the GUI and internet browser has been enabled, not only inside the own network (intranet), but also as the consumer is not at home (internet) – by using DNS and SSL/HTTPS. To increase the comfort and always maintain a full overview over the actual status of home devices, their schedules and measurements, FTUI FHEM has been installed on a tablet, as a frontend, and put into operation (Fig. 2).

B. Algorithm – target function

The optimization problem is formulated as MILP and solved using GAMS. The goal of the optimization is to satisfy energy needs of a certain household (or more of them) by scheduling the usage of electricity devices and taking into account market-driven electricity prices calculated on a day-ahead level. Due to the possibility to store certain volume of energy during the day (battery with the daily circle), it is feasible to shift its usage and perform arbitrage between different hours of a particular day. Therefore, the optimization function minimizes costs of procured electricity over the power exchange:

$$min \sum_{t=1}^{T} C_t \times \frac{1}{4} \times (PX_t^{buy} - PX_t^{sell}) \qquad (1)$$

where positive variables PX_t^{buy} and PX_t^{sell} represent the energy bought, respectively sold, at the day-ahead energy market in the particular time horizon t. The input time series, C_t, respresent market price of electricity in the particular time horizon t. The total costs defined with the target function (1) need to be minimized by taking into account different constraints defined in (2) – (15).

C. Energy balance

First of all, the total energy balance of household needs to be zero, which means that the sum of energy bought and sold needs to be equal during the observed time horizon t:

$$PX_t^{buy} + Pbd_t = PX_t^{sell} + Pbc_t + \sum_{i=1}^{N} d_{i,t}, \forall i,t \qquad (2)$$

Where Pbc_t and Pbd_t represent the volumes of energy charged and discharged out of the battery in the time period t and $d_{i,t}$ is consumption of a certain home energy appliance i in the time period t.

D. Home energy appliances (Type 1)

Although relation between minimum up/down times and unit commitment state of the power plants has been thoroughly described in literature, such as [7] and [8], a more simplified approach to model such constraints for home automation has been implemented in optimization algorithm [9]. This approach requires to define initial operating state of a unit.

As the first constraint, minimal time that represent consecutive time intervals during which home appliance i needs to run (once it has been started), is defined as MUT_i. This limitation for the optimization process is necessary, for example, to ensure that once the washing machine is started, it is kept running for at least three consecutive hours:

$$u_{i,t} \ge \sum_{t'=t+1-MUT_i}^{t} v_{i,t'} \quad , \forall i,t \qquad (3)$$

A similar limitation, defined as the minimum down time MDT_i, has been introduced to avoid constant on/off state change of a certain home appliance i:

$$1 - u_{i,t} \geq \sum_{t'=t+1-MDT_i}^{t} w_{i,t'} \quad , \forall\, i, t \qquad (4)$$

The additional binary equation (5) is used to define relation between minimum up time and minimum down time equations:

$$\mathrm{u}_{\mathrm{i,t-1}} - \mathrm{u}_{\mathrm{i,t}} - v_{i,t} - w_{i,t} = 0 \quad ,\forall i, t \qquad (5)$$

where $\mathrm{u}_{\mathrm{i,t}}$ is a binary variable that determines on/off state of an appliance i in the time interval t; $\mathrm{v}_{\mathrm{i,t}}$ is a binary variable that determines start-up status of appliance i in time interval t (in case that the appliance has been turned on it is 1, otherwise 0); $\mathrm{w}_{\mathrm{i,t}}$ is a binary variable that determines shut-down status of appliance i in time interval t (in case that the appliance has been turned off it is 1, otherwise 0). To ensure that an appliance is connected to the grid for at least certain period of time, minimum powered time ($Tmin_i$) parameter for a particular appliance i has been defined:

$$\sum_{t=1}^{h} \mathrm{u}_{\mathrm{i,t}} \geq \mathrm{Tmin}_{\mathrm{i}} \quad , \forall\, \mathrm{i, t} \qquad (6)$$

Minimal and maximal consumption of an appliance i in the time interval t is limited with (7):

$$0 \leq \mathrm{d}_{\mathrm{i,t}} \leq \mathrm{d}_{\mathrm{i,MAX}} \;, \forall\, \mathrm{i} \qquad (7)$$

Consumption of a certain home energy appliance i in the time period t, given as $\mathrm{d}_{\mathrm{i,t}}$, has been defined for type 1 of home energy appliances as discrete variable:

$$\mathrm{d}_{\mathrm{i,t}} = \mathrm{d}_{\mathrm{i,MAX}} \times (\, \mathrm{u}_{\mathrm{i,t}} - \mathrm{w}_{\mathrm{i,t}}) \;, \forall\, \mathrm{i, t} \qquad (8)$$

On top of that, a user can specify the earliest period of time when a certain energy appliance i could be started ($\mathrm{Tstart}_{\mathrm{i}}$), respectively the latest time period when a certain energy appliance i needs to finish its work ($\mathrm{Tend}_{\mathrm{i}}$).

E. *Home energy appliances (Type 2)*

Special type of home devices are those that are dependent on the room temperature. Such a case is control of the air conditioning system output during the summer months, while maintaining the indoor air temperature within a certain predefined range:

$$T_t^{in} = \varepsilon \cdot T_{t-1}^{in} + (1-\varepsilon)\left(T_{t-1}^{out} - 5\mu \frac{\mathrm{d}_{\mathrm{i,t}}}{9A}\right), \quad \forall\, \mathrm{t} \qquad (9)$$

In comparison to modelled approach proposed in [10], outside temperature is assumed not to be given as a constant parameter but rather as variable during the day. The level of comfort in a room is determined by defining lower and upper limit for indoor temperature:

$$T_{MIN}^{in} \leq T_t^{in} \leq T_{MAX}^{in}, \; \forall\, \mathrm{t} \qquad (10)$$

where $\mathrm{T}_{\mathrm{MIN}}^{\mathrm{in}}$ is defined as $\mathrm{T}_{\mathrm{t}}^{\mathrm{in}}$-$\mathrm{T}_{\mathrm{IDEAL}}^{\mathrm{in}}$ and $\mathrm{T}_{\mathrm{MAX}}^{\mathrm{in}}$ as $\mathrm{T}_{\mathrm{t}}^{\mathrm{in}} + \mathrm{T}_{\mathrm{IDEAL}}^{\mathrm{in}}$. In such a way, consumer determines deviation from the optimal indoor temperature, which needs to be maintained during all time horizons.

F. *Battery operation*

For the mathematical modeling of battery, two additional binary variables bc_t and bd_t have been introduced. If the battery is in charging mode in the timeframe t, bc_t is equal to 1, and 0 if this is not the case. For the variable bd_t it is the other way around - if the battery is in discharging mode in the timeframe t, bd_t is equal to 1, and 0 if this is not the case. In order not to allow charging and discharging of the battery in the same time interval, following equation (11) is introduced:

$$bc_t + bd_t \;\leq 1, \; \forall t \qquad (11)$$

The battery can be charged and discharged in time interval t by taking into account maximal charge and discharge limits expressed with (12) and (13):

$$Pbc_t \leq Pbc_t^{max} \times bc_t \quad , \; \forall t \qquad (12)$$

$$Pbd_t \leq Pbd_t^{max} \times bd_t \quad , \; \forall t \qquad (13)$$

Volumes of energy charged ($\mathrm{Pbc}_{\mathrm{t}}$) and discharged ($\mathrm{Pbd}_{\mathrm{t}}$) in the time period t are positive continuous variables. It is assumed that the battery is charged and discharged following the linearization function (14) which also considers different efficiencies for battery charge (γ_{bc}) and battery discharge (γ_{bd}):

$$Eb_t = Eb_{t-1} + \gamma_{bc} \times Pbc_t - \gamma_{bd} \times Pbd_t \quad , \; \forall t > 1 \quad (14)$$

$$Eb^{min} \leq Eb_t \leq Eb^{max} \;, \; \forall t \qquad (15)$$

where Eb_t is the energy stored in battery in the time period t and Eb^{min} and Eb^{max} represent the minimum and maximum energy limits of the battery. With the equation (15), lower and upper energy bounds of a battery have been limited.

IV. Case Study

A. *Description of input parameters*

With the AC-DC inverter in place and actual efficiency of rechargeable lithium-ion battery, the round trip efficiency of 87% has been assumed. The battery is charged (respectively discharged) with the constant power of maximal 1.5 kW (respectively maximal 0.9 kW), having the total capacity limit of 3.6 kWh and operating on the constant temperature of 25 deg C. At the beginning of optimization period, it was considered that battery is at its lower charging level, being 0.5 kWh. In order to have the equal ground for comparison of case studies, the same battery charge level has been assumed for the last period of time. The lower charging level is imposed due to the safety reasons, as especially valid for lithium batteries. It was assumed that battery could change not only its operating points in the consecutive hours, but also operation state, from charge to discharge and vice versa. No limitation has been given in this respect, which could lead to constant changes of operation state in the hours of volatile market price signals, such as consequent periods of low and high prices. If the limitation of minimal number of hours that battery needs to spend in a certain operation state is given, this would improve battery lifetime but would influence financial savings on optimized energy usage.

Comfort level of indoor temperature is defined to be 23 deg C and a user set its preference for the deviation range of +/- 2 deg C that needs to be maintained during the day. This temperature is also influenced by the development of outdoor temperature. Limitations of distribution system operator have been considered by defining the maximal power that could be withdrawn by a household from the electricity network (8 kW). It was assumed that all home devices start from the idle state, which is the initial condition for optimization process.

B. Discussion of results

To demonstrate how the unit commitment of smart home solution along with the deployment of battery would work in practice and which advantages it would bring, three case studies have been performed (Table I) using the architecture explained in the chapter III A. For all of them, a set of home devices that include lights, air conditioner, refrigerator, washing machine, washer-dryer, dishwasher and cooker, has been scheduled by taking into account constraints described in the model and explained in the chapter III C-F. Such limitations include the minimal powered time as well as minimal consecutive time intervals during which a home appliance needs to operate by consuming electricity (respectively stays shut down) once it has been started (respectively disconnected from the network).

TABLE I. SMART HOME – ANALYZED CASE STUDIES

Device Name	*Case 1*	*Case 2*	*Case 3*
Total energy stored (battery) [kWh]	-	26.74	26.74
Total energy produced (battery) [kWh]	-	26.43	26.43
Energy bought at the power exchange [kWh]	-	117.21	108.16
Energy sold at the power exchange [kWh]	-	25.12	15.88
Consumption of the devices [kWh]	91.79	91.79	91.79
Total energy costs [EUR]	1.82	1.65	1.25

On top of that, a user can express its preference and define the additional constraints such as the earliest point of time when a certain device could start its operations, respectively the latest time when it needs to stop its work (Table II). For example, one could specify that cooker needs to be scheduled between 11am and 3pm and it needs to work for at least 1 hour by consuming 4 kWh in total.

All devices, except air conditioner, are scheduled either to work with the nominal power or to be completely disconnected from the electricity network. Only the air conditioner needs to be scheduled in relation to outdoor and indoor temperature and could be operated continuously, up to 3.5 kW.

In the Fig. 3, optimized commitment of the air conditioner has been shown, along with the room temperature (red line) which has been maintained within the predefined limits. The indoor temperature is influenced by the weather changes during the day, given over outdoor temperature (blue line). As it could be seen from the Fig. 4, the device works with its maximal power (3.5 kW) in the early morning when the day-ahead energy price is the lowest one (below 20 cEUR/kWh). This operational schedule brings the room temperature to the lowest comfort level being allowed (21 deg C). The cooling-down of the room is done with the goal to reduce the total energy costs, as in the following hours (8 a.m. and 9 a.m.) day-ahead energy market price peaks to almost 140 cEUR/kWh (the first morning peak) and device is completely being disconnected from the network. Due to the very tight band for maximal and minimal allowed room temperature, but also due to the strong increase of the outdoor temperature in period from 7a.m. to 4 p.m., it is necessary to run air conditioner in the periods of higher prices and to maintain the indoor temperature at the maximal allowed level (25 deg C). In a case that the smart home system doesn't contain battery system to store the energy withdrawn from the market in low-priced hours (Case 1), a user would be fully exposed to the day-ahead energy market price achieved in those hours. With the optimal commitment of the battery, the total costs could be reduced even further (Case 2 and Case 3).

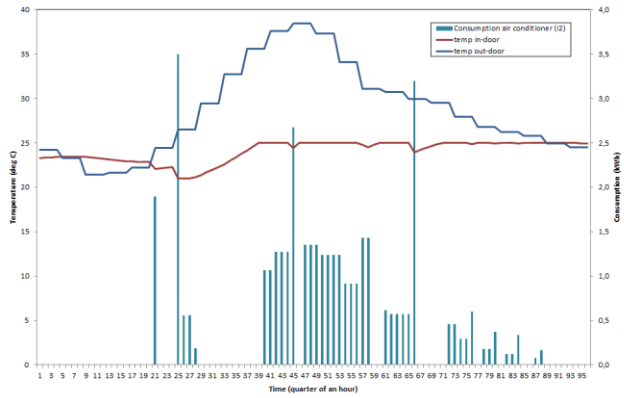

Fig. 3. Commitment schedule of the air conditioner - i2 (Case 2)

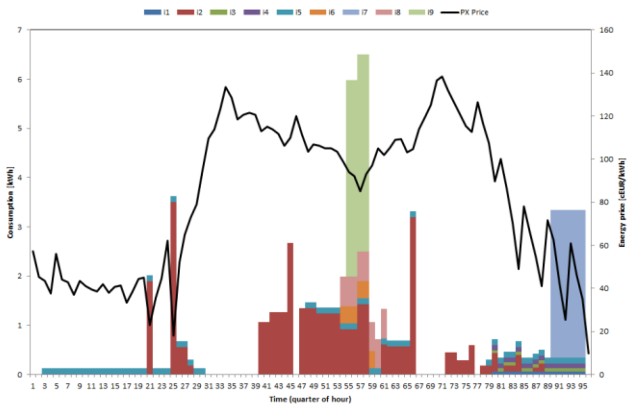

Fig. 4. Commitment schedule of all home devices (Case 2)

In the Fig. 5 one could observe that there is neither scheduled operation of home energy devices nor charging of the battery during the second price peak period. Although large number of the devices need to operate in the afternoon hours, due to their flexibility defined by minimal consecutive time intervals during which a home appliance needs to run or stay disconnected from the network as well as due to minimal powered time of a device, it is possible to avoid the second (afternoon) price peak and to minimize the total costs of procured electricity.

TABLE II. MAIN FEATURES OF HOME APPLIANCES

Symbol	***Device Name***	***Type***	***Power [kW]***	***Tmin***	***MDT***	***MUT***	***Tstart***	***Tend***
i1	Light bulb 1	1	0.06	16	4	4	72	96
i2	Air conditioner (i2)	2	< 3.5	-	1	1	0	96
i3	Light bulb 2	1	0.06	16	4	4	72	96
i4	Light bulb 3	1	0.1	16	4	4	72	96
i5	Refrigerator	1	0.12	64	1	4	0	96
i6	Washing machine	1	0.35	6	1	4	48	72
i7	Washer-dryer	1	3	6	1	4	72	96
i8	Dishwasher	1	0.6	8	1	8	28	72
i9	Cooker	1	4	4	1	4	44	60

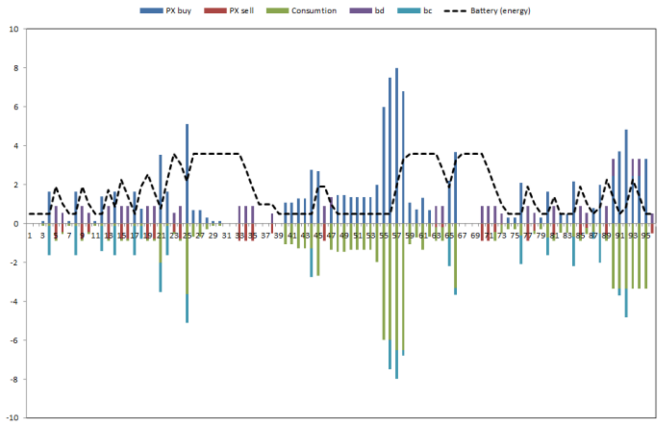

Fig. 5. Deployment of the battery system (Case 2)

As it is shown in the Fig. 4, only two devices, namely air conditioner (i2) and refrigerator (i5), could be optimized during the entire day. A sudden drop of price in the afternoon hours (12 a.m. – 4 p.m.) is used to schedule cooker (i9) and dishwasher (i8). Most of other home appliances are scheduled in the early or late evening hours, once the market prices tend to decrease. Without considering a possibility to use battery system (Case 1), total daily energy procurement costs would be 1.82 EUR. By connecting a battery to the smart home and deploying it optimally during a day (Case 2), total costs in the analyzed example could be reduced down to 1.65 EUR, or for -9.3% in comparison to Case 1. The battery has been charged and discharged during the day by taking into account lower (0.5 kWh) and upper (3.6 kWh) energy limits. As one would expect, the algorithm tends to use energy price drops during the day to charge the battery and to use the accumulated energy once the price reaches its peaks (Fig. 5). Battery has been optimally deployed during the night hours to reduce costs of the energy consumption caused by refrigerator, as it is the only device that needs power supply for 16 hours per day. It needs to be noted that a minimal charge time, respectively discharge time, is not imposed on the battery which would not allow a constant change of charge (resp. discharge) status. An alternating sequence of charge (resp. discharge) states could be especially visible in the early morning and late evening hours as the market energy price is very volatile. In the third analyzed situation (Case 3), total savings with the battery system have been calculated by not considered user preferences about the earliest possible start time of device operation, respectively the latest stop of its operation. In such a way, total costs were down to 1.25 EUR, which means a decrease for -31.3% in comparison to Case 1.

V. CONCLUSIONS

In this paper, we used MILP to define and solve optimal scheduling of home appliances by taking into account battery system with a daily circle for energy storage and constraints imposed by a household user and system operator. Total costs savings are calculated for two cases with the installed battery system and compared with the starting base case scenario (no battery available). We found that the daily energy cost reduction could be achieved using the smart home system in combination with the battery as a storage capacity.

To draw further conclusions, this work could be expanded to consider minimal consecutive charging (and discharging) periods of the battery system and/or smart home investment costs, but also to perform a detailed cost-benefit analysis which would include a longer period of system deployment (e.g. one year).

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